On the first Townsend coefficient at high electric field

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Abstract

For the first time it is shown and experimentally confirmed that gas gain in wire chambers at very low pressure become higher on thicker wires at the same applied high voltage. This is a consequence of the fact that the first Townsend coefficient at high reduced electric field depends almost entirely on the electron's mean free path.

Key words: Townsend coefficient, gas gain, low pressure

1 Introduction

An electron drifting between two points r_1 and r_2 under the influence of an electric field gains energy and produces secondary electrons due to inelastic collisions. When attachment, photoproduction and space charge effects are negligible, the multiplication factor M over that path is expressed by

$$lnM = \int_{r_1}^{r_2} \alpha(r)dr, \tag{1}$$

where $\alpha(r)$ is the first Townsend coefficient. The Townsend coefficient is a function of reduced electric field strength $S= E/P$, i.e. $\alpha/P = f(E/P)$. There are several forms of first Townsend coefficient [1]. Most of them satisfactorily describe experimental data

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on some ranges of electric field with correctly determined parameters. The generalized form of reduced first Townsend coefficient is given by the expression

$$\frac{\alpha}{P} = Aexp(\frac{-BP}{E}), \tag{2}$$

where A and B are parameters depending on gas type and electric field range. It has been shown [2] that this generalized form of first Townsend coefficient is a satisfactory description of experimental data in a wide range of electric field. This form of the first Townsend coefficient will be used hereafter as well.

2 Townsend coefficient at high reduced electric field

Let λ represent an electron's path between two consecutive collisions, λ_m is the mean free path in the field direction between two collisions with atoms, and λ_i is an ionization path, i.e. the distance required to gain enough energy for successful ionization. It is obvious that for each gas λ_m and λ_i depend on pressure. λ_i is a function of local electric field as well.

In gases, distances between two consecutive collisions of electrons and atoms have a Poisson distribution. In general, the mean free path in gases is defined as

$$\lambda_m = \int_0^\infty xp(x)dx = \frac{1}{n\sigma}, \tag{3}$$

where n is the gas density (number of atoms per unit volume) and σ is the cross section for electron collision with atoms. Generally, cross section is a function of electron velocity (energy) $\sigma = \sigma(v)$.

The first Townsend coefficient in the generalized form expressed by eq.(2) is valid at relatively low values of electric field. By definition the first Townsend coefficient is the number of secondary electrons produced per unit length. When distances between two collisions become greater than the electron path required to gain enough energy to ionize atoms, i.e. $\lambda \geq \lambda_i$, the first Townsend coefficient is expressed very simply as

$$\alpha = \frac{1}{\lambda} \tag{4}$$

Therefore the first Townsend coefficient can be expressed as a combination of two components:

$$\begin{aligned} \alpha_1 &= APexp(\frac{-BP}{E}), \lambda < \lambda_i \\ \alpha_2 &= 1/\lambda, \lambda \geq \lambda_i \end{aligned} \tag{5}$$

with the general gas multiplication factor of eq.(1) expressed now as

$$lnM = \int_{r_1}^{r_2} (\alpha_1(r) + \alpha_2(r))dr. \tag{6}$$

The first term in eq.(5) describes events when there are several collisions between two consecutive ionizations. The second one should be taken into account for the cases when there are no elastic collisions between two consecutive ionizations.

Consider the hypothetical simplified case when free paths between two collisions of electrons with atoms are constant at a given pressure and equal to $1/n\sigma$. In that case λ in eq.(5) should be replaced by λ_m.

An electron drifting at electric field E over the distance λ_m gains energy $eE\lambda_m$. For any gas under all conditions there exists an electric field strength E_m where the electron gains enough energy to ionize the gas over the path λ_m, i.e.

$$eE_m\lambda_m = I_0, \tag{7}$$

where I_0 is the gas ionization potential. For this case eq.(5) could be re-written as

$$\begin{aligned} \alpha_1 &= APexp(\frac{-BP}{E}), E < E_m \\ \alpha_2 &= n\sigma, E \geq E_m \end{aligned} \tag{8}$$

3 Applications to PPAC and wire chambers

There are very interesting consequences from the last equations both for PPAC and wire chambers.

In case of the PPAC the electric field is homogenous and the multiplication factor over the cathode-anode distance is expressed, according to eq.(8), as

$$lnM = \int_{a}^{c} (APexp(\frac{-BP}{E})dr \tag{9}$$

when $E < E_m$ or

$$lnM = \int_{a}^{c} (n\sigma)dr \tag{10}$$

when $E \geq E_m$.

This means that in the simplified case of constant free paths when $\lambda \equiv \lambda_m$ there is a limit of an electric field strength E_m where further voltage increase does not increase gas multiplication (assuming that electric field and gas pressure have values where electrons gain energy which is not enough to produce double electron ionizations). This field E_m is given by eq.(7) with λ_m defined by eq.(3).

The same limitation could affect operation of gas electron multipliers (GEM) at low pressure, although the electric field in the GEM is non-uniform.

At standard conditions for an ideal gas the number of atoms in a unit volume is $n = 2.687 \cdot 10^{19}\ cm^{-3}$ (Loshcmidt number). Generally, the cross section is a function of electron energy and the typical value is the order of $10^{-15}\ cm^2$ [3]. Typical ionization potentials for different gases are in the range 10-15 eV. Substituting values for σ, n and I_0 into the eq.(7) gives $E_m \simeq 270 - 400\ kV/cm$ which is far beyond a reachable value. However, the situation changes at low pressure. At a gas pressure of 20 $Torr$, $n \simeq 7 \cdot 10^{17}\ cm^{-3}$, and eq.(7) gives an electric field value in the range $E_m \simeq 7 - 10.5\ kV/cm$, or $S_m \simeq 350 - 520\ V/cm \cdot Torr$ in terms of reduced electric field. These values are in the range where PPACs are used at low pressure [4].

More important consequences from the consideration of eq.(8) appear for wire chambers. The electric field in a cylindrical wire chamber is defined as

$$E = \frac{V}{rln(b/a)}, \tag{11}$$

where a and b are wire and cathode radii, and r is the distance from the wire center. The electric field strength has a maximum value on the wire surface and sharply drops off moving away from the wire. As result, at atmospheric pressure the gas gain mainly takes place within 3-5 wire radii.

Real gas amplification starts when the electric field becomes greater than some critical level E_c. This critical value is characteristic of the gas and is in the range 40-70 kV/cm for different gases at atmospheric pressure or $S_c \simeq 50 - 90\ V/cm \cdot Torr$ in terms of reduced electric field. Reduced critical electric field $S_c = E_c/P$ is a constant for a given gas and independent of its pressure.

In wire chambers with the same geometry and different anode wire diameters at atmospheric pressure one needs to apply much lower voltage to chambers with a thin wire in order to get the same gas gain. At lower pressure the voltage difference required to get the same gas gain between two different anode wire sizes becomes smaller. Now recall that there exists an electric field E_m where an electron gains enough energy between two consecutive collisions to ionize the gas. Although the field near the wire is non-uniform, for simplicity one can consider an average electric field over the path between two consecutive collisions. We found that for gases at standard conditions this field is in the range $E_m \simeq 270 - 400\ kV/cm$ or $S_m \simeq 350 \div 520\ V/cm \cdot Torr$ in terms of reduced electric field. Like a critical reduced electric field strength S_c, the S_m is a constant for a given gas and is independent of pressure.

Let us consider two single wire chambers with cathode diameters 10 mm and wire diameters 10 μm and 50 μm. Fig.1 presents reduced electric field value near the 10 μm and 50 μm wires as a function of distance from the wire surface. 1200 V applied

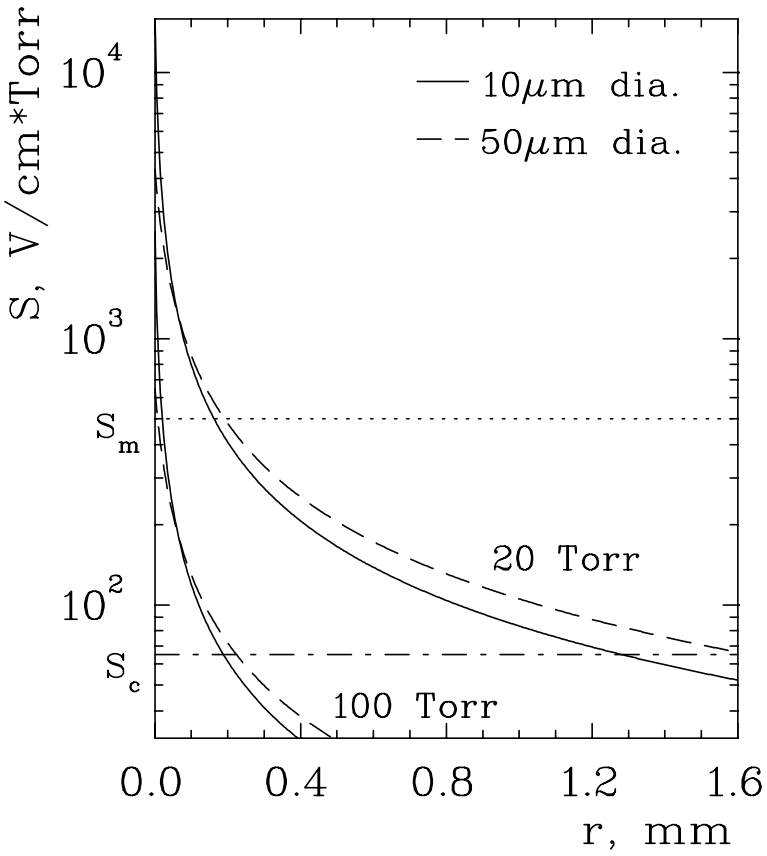

Fig. 1. Reduced electric field strength as function of distance from the wire surface for single wire chambers with cathode diameters 10 *mm* and wire diameters 10 and 50 μm. Both wires are at 1200 *V* at 100 *Torr* and at 900 *V* at 20 *Torr*.

to both wires at pressure 100 *Torr* and 900 *V* at 20 *Torr*. We will use typical values of $S_c = 65\ V/cm \cdot Torr$ and $S_m = 500\ V/cm \cdot Torr$ in further consideration. These values S_c and S_m are shown on the figure as well. For each wire let r_c be the point where $E = E_c$, i.e. the point where the gas avalanche starts, and r_m be the point where $E = E_m$. Fig.1 shows that at 100 *Torr* the avalanche starts about 0.2 *mm* away from the 50 μm wire surface and about 0.18 *mm* away from the 10 μm wire. At 20 *Torr* these values are 1.6 *mm* and 1.3 *mm* respectively.

With the same chamber geometry and applied voltages the electric field strength is much higher on the surface of the thin wire. It drops off faster on the thin wire and eventually the field strength becomes higher near the thick wire as can be seen on fig.1. Fig.2 shows details of the same field strengths with a linear vertical scale.

The gas gain on each wire is defined by eq.(1) by integrating the first Townsend coefficient from the avalanche start point r_c to the wire surface r_a. Considering again the simplified case where free paths are constant and $\lambda \equiv \lambda_m$ we can divide that integral into two parts. One part includes the path from r_c to r_m where $S < S_m$, and

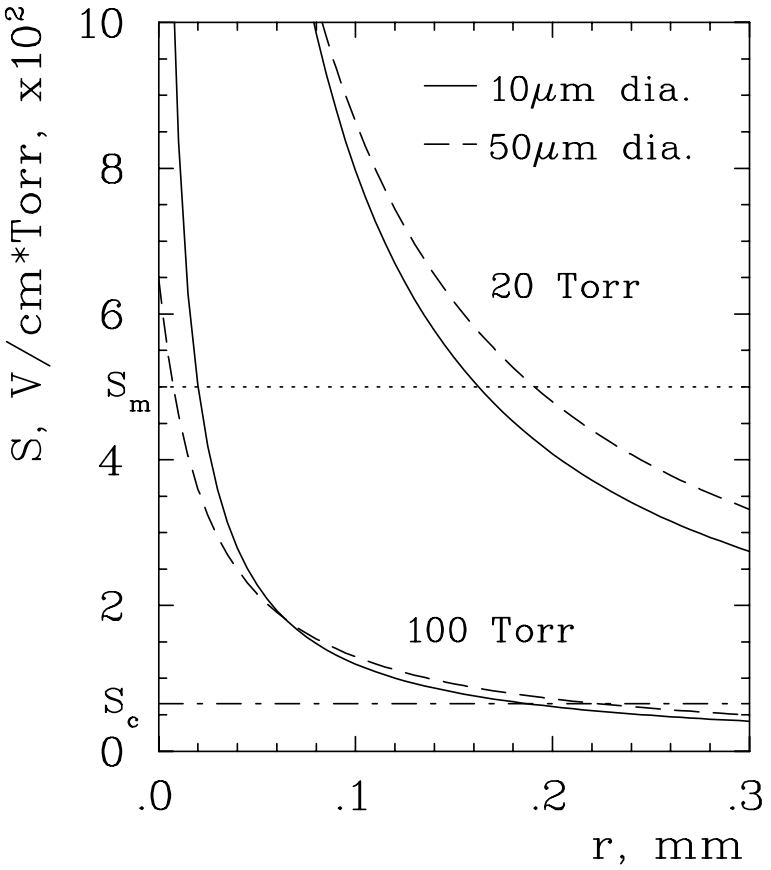

Fig. 2. Details of reduced electric field strength from fig.1 within 0.3 *mm* from the wire surfaces. Note the linear vertical scale. Both wires are at 1200 *V* at 100 *Torr* and at 900 *V* at 20 *Torr*.

the second one is from r_m to the wire surface r_a. So, the gas gain is defined as

$$lnM = \int_{r_c}^{r_m} \alpha_1(r)dr + \int_{r_m}^{r_a} \alpha_2(r)dr, \tag{12}$$

where $\alpha_1(r)$ and $\alpha_2(r)$ are defined by eq.(8).

In the simplified case $\lambda \equiv \lambda_m$ at reduced electric field strength $S > S_m$ between any two consecutive collisions with atoms, electrons gain enough energy to ionize the gas. As result, the first Townsend coefficient becomes independent of the electric field strength (although this is not exactly correct since the cross section σ is a function of electron energy).

Let us consider in detail the electric field strengths at a pressure of 100 *Torr* and 1200 *V* an applied voltage. The gas gain starts earlier on the 50 μm wire. However, the field is very weak here and the contribution to the total gain is insignificant. Eventually the electric field strength becomes higher on the 10 μm wire. Everywhere

after the field crossing point the first Townsend coefficient for 10 μm wire is higher or equal (after electric field strength on 50 μm wire exceed the E_m value) to that on the 50 μm wire. As a result, the gas gain on the 10 μm wire is higher than on the 50 μm wire at this pressure.

At 20 *Torr* pressure and 900 *V* applied to both chambers the situation is different. As usual, at distances far from the wire surfaces the electric field strength is higher on the thick wire and gas gain starts earlier on that wire. However unlike the previous case the electric field strength lines cross above the $S_m = 500\ V/cm \cdot Torr$ line. This means that the first Townsend coefficient is higher on the 50 μm wire everywhere until the electric field strength near the 10 μm wire reaches the value $S = S_m = 500\ V/cm \cdot Torr$. After that point both wires have the same first Townsend coefficients. As a result, at pressure 20 *Torr* and 900 *V* applied high voltages the gas gain on the 50 μm wire is higher than that on 10 μm wire.

We considered the simplified case with constant electron free path lengths. In real gases the electron free paths have a Poisson distribution with mean value expressed by eq.(3). Also, the ionization cross section σ_i is only a fraction of the total cross section σ at electron energies just above ionization level. The first Townsend coefficient at each point can be defined through the mean free path [5] multiplying it by the probability that the path is longer than the local $\lambda_i(r)$ and by the ratio of ionization cross section to the total cross section σ_i/σ, i.e.

$$\alpha(r) = \frac{\sigma_i}{\sigma}\frac{1}{\lambda_m}exp(-\frac{\lambda_i(r)}{\lambda_m}), \tag{13}$$

The first term in eq.(8) should be multiplied by the probability that path is shorter than the local ionization path $\lambda_i(r) = I_0/eE(r)$. Finally, eq.(8) transforms to

$$\alpha(r) = APexp(\frac{-BP}{E})(1 - exp(-\frac{\lambda_i(r)}{\lambda_m})) + \frac{\sigma_i}{\sigma}\frac{1}{\lambda_m}exp(-\frac{\lambda_i(r)}{\lambda_m}) \tag{14}$$

or, using definitions of λ_i and λ_m

$$\alpha(r) = APexp(\frac{-BP}{E(r)})(1 - exp(-\frac{I_0 n\sigma}{eE(r)})) + n\sigma_i exp(-\frac{I_0 n\sigma}{eE(r)}) \tag{15}$$

Unlike the simplified case where paths between two consecutive collisions of electrons and atoms are constant, in real gases there will be no sharp transition from the first term of the Townsend coefficient to the second one. The first term in eq.(15) will dominate at relatively low reduced electric field. This term vanishes at high reduced electric field and the Townsend coefficient will almost entirely depend on the second term.

Values S_c and S_m are constant for each gas and vary from gas to gas. As a result, different gases will have different pressures and applied high voltages where gas gain becomes higher on the thick wire at the same chamber geometry and applied voltages.

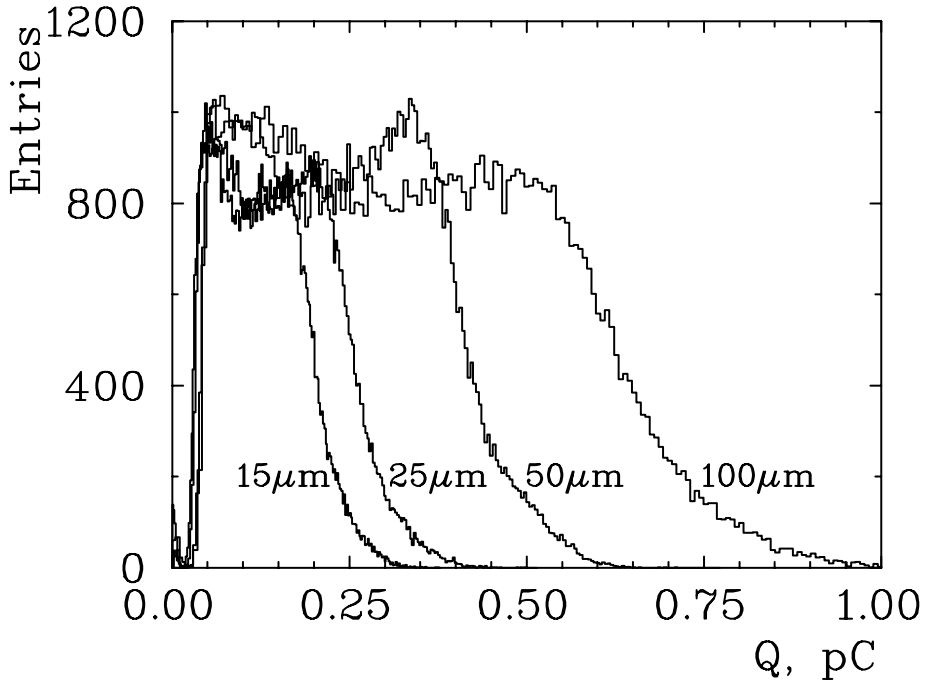

Fig. 3. Measured charge spectra on single wire chambers with cell size 12x12 *mm* and 15, 25, 50 and 100 μm diameter anode wires filled with pure $iso - C_4H_{10}$ at 12 *Torr*. All chambers are at 800 *V* and irradiated with ^{55}Fe x-rays. The bigger the wire diameter, the higher the gas gain.

A set of measurements of gas gain in single wire chambers with 12x12 mm cell cross sections with different wire diameters (15, 25, 50 and 100 μm) have been made in order to check the gas gain behaviour. Chambers were filled with pure $iso - C_4H_{10}$ at pressures either 92, 52, 32 or 12 Torr and irradiated with ^{55}Fe x-ray particles. Most of the electrons released by the photoabsorption process in the gas volume, leave the chamber cell. A small fraction of them lose their entire energy inside of the cell and give full signals and these photoabsorption peaks were used to calculate gas gain. The intensity of these full photoabsorption peaks drops with decreasing gas pressure due to the increasing electron range in the gas. Taken data show that at 92 Torr gas gain on thinner wires are higher at the same applied voltage. However, gain become higher on the 50 μm wire compared with that on 15 and 25 μm wires already at 32 Torr at the same applied high voltage on wires. Results clearly demonstrate that a decrease of gas pressure leads to a higher gas gain on thicker wires compared to that on thinner ones. Detailed results of these measurements will be published separately [6].

Figure 3 presents one result from these measurements, namely the charge spectra taken from single wire chambers at a pressure of 12 *Torr*, with 800 *V* applied to all chambers. Electrons released by the photoabsorption of ^{55}Fe x-ray particles have a range of about 45 *mm* in pure $iso - C_4H_{10}$ at 12 *Torr*. Most of them leave the chamber cell before losing all of their energy and there is no evidence of pho-

toabsorption peaks. The resulting charge spectra from the chambers have continuous distributions. However, all chambers have the same initial ionization distribution and the edges of the spectra do indicate the gas gain on each wire. The figure clearly demonstrates that in pure $iso - C_4H_{10}$ at 12 $Torr$ and applied 800 V the thicker wires have higher gas gain. Taking a collected charge on each wire at the half maximum on the edge of the spectra as a reference gives the ratio of gas gains on all these wires as $M_{15} : M_{25} : M_{50} : M_{100} \simeq 1 : 1.35 : 1.85 : 3.15$.

It should be noticed that a chamber simulation program Garfield [7] with Magboltz [8] interface shows a similar tendency on gas gain at low pressure. Although Garfield overestimates the first Townsend coefficient and as a result gas gain, one can compare the gas gain ratios on different diameter wires. The Garfield estimation of avalanche sizes due to single electrons on the same wire chambers under the same conditions as in the above mentioned example gives the ratio of gas gains $M_{15} : M_{25} : M_{50} : M_{100} \simeq 1 : 2.18 : 4.79 : 6.47$.

4 Conclusion

We have shown that at high reduced electric field, where drifting electrons gain enough energy to ionize gas atoms in one free path, the first Townsend coefficient should almost entirely depend on the electron's mean free path. The generalized formula for the first Townsend coefficient at high reduced electric field should include the measured gas data as parameters.

For the first time, it is experimentally shown that gas gains in wire chambers at very low pressure become higher on thicker wires at the same applied high voltage. The reason for this is that at high reduced electric field the first Townsend coefficient has weak dependence on the electric field. Thinner wire has much higher electric field in the vicinity of the wire surface at the same applied high voltage. However, the first Townsend coefficient here stays almost the same as on the thick wire. Thicker wires should be used in wire chambers operating at very low pressure where scattering on the wires is not critical.

In PPAC's at some value of reduced electric field strength for the simplified case of constant free paths there is a limit for gas gain, which is defined by gas density, i.e. electron's free paths. These values vary from gas to gas and are in the range $S_m \simeq 340 - 500\ V/cm \cdot Torr$. In real gases the free paths have a Poisson distribution, and there will be no sharp transition when the electric field reaches the value S_m. It should asymptotically approach to its limit instead. One can expect similar behaviour in GEMs operated at low pressure.

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